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AN ANALYSIS OF THE EFFICIENCY OF ACCEPTANCE SAMPLING PLANS

A Thesis

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By

Tee Hansford Hiett, Jr.

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AN ANALYSIS OF THE EFFICIENCY

OF ACCEPTANCE SAMPLING PLANS

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SUMMARY

The purpose of this study was to present a method which would readily permit an analysis of the efficiency of acceptance sampling plans as used in quality control.

The method of approach was to first provide a definition of efficiency. Efficiency was defined as the ability to discriminate between good and bad quality lots. To permit further analysis of the risk involved, it was necessary to define two additional areas of efficiency; one, producer's efficiency which was defined as the ability of the sampling plan to accept submitted lots which should be accepted and two, consumer's efficiency which was defined as the ability of the sampling plan to reject submitted lots which should be rejected.

This information is illustrated by the operating characteristic (OC) curve for a given sampling plan. By assuming an infinite lot size, the number of points through which the OC curve is drawn increases and approximates a continuous distribution. By further assuming that the quality of incoming lots is unknown and likely to be any value from zero to one, it was possible to determine the efficiencies by the following methods.

The producer's efficiency was determined by the ratio of the area below the OC curve and left of the acceptable quality level (AQL) to the area which would be to the left of AQL if the sampling plan were perfect.

The consumer's efficiency was determined by the ratio of the area above the OC curve and right of AQL to area which would be to the right of AQL if the sampling plan were perfect.

The efficiency was determined by adding the area below the OC curve and left of AQL to the area above the OC curve and right of AQL.

The Poisson distribution was used to measure the areas required.

The sampling plans to be evaluated were taken from Military Standard 105A, "Sampling Procedures and Tables for Inspection by Attributes". The sample sizes ranged from 20 to 1000, AQL and acceptance number were selected to be comparable to those used in Military Standard 105A.

The results are presented in graphs from which it is possible to determine the change in efficiency resulting from change in acceptance number, AQL, and sample size or any combination of these factors.

CHAPTER I

INTRODUCTION

The amount of time and work involved in 100 per cent inspection (inspection of each item in a given lot) or perhaps destructive testing demand that samples be taken to examine the specification of the lot. "Sampling may be described as a process for estimating some measurable function of the quality of a certain quantity of an item by examination of a portion of the quantity in question". (1, p. 21) No sampling plan will give this information for a single lot but a good sampling plan will give an average, over-all lot quality estimation for a series of sampled lots.

Until just before World War II many statements resulting from a study of samples were little more than guesses. A rapid development of sampling theory took place at that time as a result of industry's need for better sampling methods. During the war this theory was expanded and tested. This development has continued and today, sampling plans based on sound theory are readily available.

Sampling inspection has two major functions: one, to provide a check on production to aid in the reduction of defectives produced, and two, to aid in reducing the number of excessively defective lots which are accepted or shipped out. Each of these functions are to be performed with a minimum amount of inspection.

There are two basic types of sampling plans, inspection by attributes and inspection by variables. In attribute inspection the individual pieces of the sample are classified as either defective or not defective. In inspection by variables, the degree of variability and the average of a group of individual observations are used to predict the acceptability of the lot. This thesis is concerned with attribute sampling.

There are various acceptance sampling plans in use but these resolve basically to three principle types; single sampling plans, double sampling plans and sequential sampling plans.

A single sampling plan will accept the lot if the number of defective items found in a random sample taken from the inspection lot does not exceed a predetermined number. A plan can be illustrated as follows: Take a random sample of 75 (n) items from an inspection lot of 1000 (N) items and inspect each item in the sample. If the number of defectives (x) in the sample does not exceed a predetermined acceptance number (c) of three, accept the entire lot. It might be well to state at this point that the symbols and terms used in this thesis are basic and consistent with most leading textbooks in the field of Quality Control.

The number of defective items found in a sample taken at random is a matter of probability. In a situation where the sample comprises a significant portion of the lot from which it is drawn, the distribution of the sample results can be represented by the hypergeometric distribution. Here the probability of a defective item being selected may change materially as the sample is drawn. If an inspection lot contains N items, m of which are defective, the probability that a random sample of n items will contain c or fewer defectives (indicating acceptance of the lot) is given in terms of the hypergeometric distribution formula. (2 p. 85)

$$Pa = \sum_{x=0}^{c} \frac{\binom{N-m}{n-x}\binom{m}{x}}{\binom{N}{n}} \qquad (1)$$

where:

Pa = Probability of acceptance (Probability of c or fewer defectives in sample)

x = Number of defective items in the sample

$$\binom{N-m}{n-x} = \frac{(N-m)\,!}{(n-x)\,!\ \left[(N-m)-(n-x)\right]\,!}$$

= number of combinations of effective items in the sample that can be made from the effective items in the lot.

$$\binom{m}{x} = \frac{m\,!}{x\,!\ (m-x)\,!}$$

= number of combinations of defective items in the sample that can be made from the defective items in the lot.

$$\binom{N}{n} = \frac{N}{n\,!\ (N-n)\,!}$$

= number of combinations of all items in the sample that can be made from all items in the lot.

This method of computation is extremely difficult even with the use of tables of factorials. Fortunately, there are methods of approximating these values.

If N is over eight times as great as n, the binomial distribution serves as a very good approximation for the hypergeometric distribution. (3, p. 300) In this case the sample is not a significant portion of the lot and the probability of selecting a defective item does not change materially as the sample is drawn. The probability of acceptance (c or fewer defectives in the sample) is given in terms of the binomial distribution formula. (4, p. 808)

$$Pa = \sum_{x=0}^{c} \binom{n}{x} (p)^{x} (1-p)^{n-x} \qquad (2)$$

where:

p = per cent defective of the submitted lot.

There are tables available which give the sum of the binomial coefficients. (5)

As the sample size and the acceptance number increase, the calculations involving the use of the binomial become burdensome. However, for small p and a fairly large n, the binomial distribution can be approximated by the Poisson distribution. This is especially useful where p is less than .05 and n is greater than 20. (3, p. 301) The probability of acceptance can be determined from the Poisson distribution formula.

$$Pa = \sum_{x=0}^{c} \frac{(np)^{x} e^{-np}}{x!} \qquad (3)$$

A double sampling plan will accept the entire lot if the number of defective items (x_1) in the first sample is below a predetermined number (c_1) and rejects the lot if the number of defective items in the sample is above a larger predetermined number (c_2). If the number of defective items in the sample is between $c_1 + 1$ and c_2 inclusive another sample is taken. The lot is accepted if the cumulative number of defective items ($x_1 + x_2$) is equal to or less than c_2 and the lot is rejected if $x_1 + x_2$ is greater than c_2. The probability of acceptance can be computed by the following formula:
(6, p. 185)

$$Pa = P''(c_1, n_1) + \sum_{k = c_1 + 1}^{c_2} p(k, n_1)\, P''(c_2 - k, 2n_1) \qquad (4)$$

where p(c, d) denotes the probability of c defective items in d items and P"(c, d) denotes the probability of c or fewer items in d items.

Sequential sampling is similar to double sampling with the exception that more than two samples may be required before a decision can be made. This is usually truncated (grouped sequential sampling) and a decision to accept or reject is made after the fifth to seventh sample. The algebraic method of computing the probability of acceptance is rather lengthy and complicated. There are, however, simple arithmetic procedures which will produce adequate results if properly followed. (6, p. 190)

There are numerous other sampling plans available for either inspection by attributes or inspection by variables. One example is the "Hamilton Lot Plot Method." (7, p. 15)

Every sampling plan has an operating characteristic (OC) curve which illustrates the effectiveness of the plan. The OC curve illustrates the relationship between the per cent defective of submitted lots and the probability of acceptance (Pa). The values for this curve can be obtained from equations (1), (2) or (3) for single sampling plans; equation (4) for double sampling plans and by the method illustrated in the reference for sequential sampling plans. The ordinate of the OC curve is the probability of acceptance ranging from zero to one. The abscissa is the fraction defective as (p) of submitted lots which also ranges from zero to one. Therefore, the total area in which the OC curve is defined is equal to one.

A random sample from a submitted lot does not perfectly represent the quality of the lot from which it was taken. It follows that if successive lots of equal quality are submitted, a sampling plan will accept some of these lots and reject some lots. As illustrated in Fig. 1, if every submitted lot is 2 per cent defective, 95 per cent of the lots will be accepted and 5 per cent of the lots will be rejected. The dashed line in Fig. 1 illustrates a perfect OC curve which will accept all lots in which the quality is equal to or less than the acceptable quality level (AQL) of 2 per cent and reject all lots in which the quality is greater than 2 per cent.

A comparison between methods of sampling should be made with matched plans in which the OC curve for all plans compared are approximately the same. Shafer has prepared a table of appraisal for four sampling plans which compares eight factors of each plan and he reports, "Each system under certain circumstances is superior to all

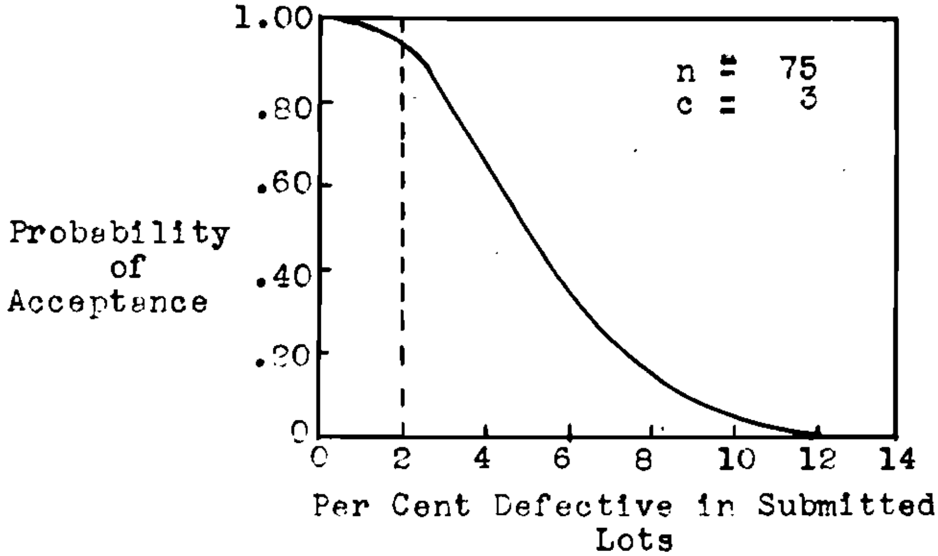

Fig. 1. Operating Characteristic Curve

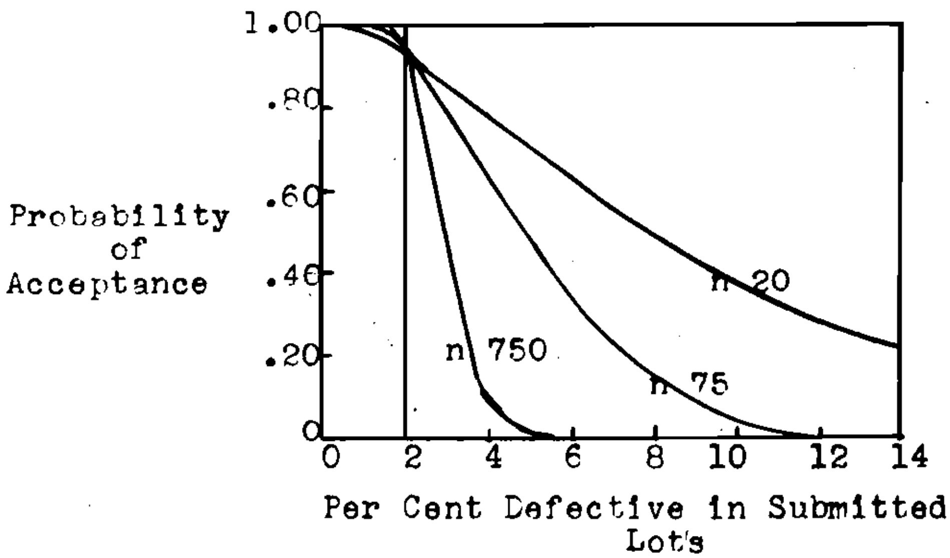

Fig. 2. OC Curve Showing Effect of Change in Sample Size

others." (8, p. 16) Peach indicates that single sampling plans have the advantage of simplicity and, if properly designed, are about as effective as double sampling plans. (9, p. 11)

To simplify the computations, this thesis is concerned only with single sampling plans. However, the results could be extended to other methods of acceptance sampling by using plans in which the OC curve matched the OC curve of the single sampling plan.

A single sampling plan can be determined by the selection of n and c. However, it is necessary to provide some insight into the protection afforded by the plan. This is usually accomplished by classifying the plan in terms of AQL or in terms of the lot tolerance per cent defective (LTPD). AQL is defined as the average fraction defective desired in accepted lots. LTPD is defined as the maximum defective tolerated in accepted lots.

Classification by the AQL establishes a per cent defective which is desired as the process average. The probability of rejection of lots in which the per cent defective is equal to the AQL is called the producer's risk (PR). It is accepted practice to let the PR equal 5 per cent. Design of sampling plans under this classification require n and c to be selected so that at the per cent defective equal to the AQL, the Pa equals one minus PR. This method of classification offers very little indication of the Pa of lots in which the quality is worse than the AQL.

Classification by the LTPD refers to a selected per cent defective above which incoming lots have a small Pa. The Pa of lots in which the per cent defective is equal to the LTPD is called the consumer's risk (CR). It is accepted practice to let the CR equal 10 per cent. Design of sampling under this classification require that n and c be selected so that at a per cent defective equal to the LTPD, the Pa equals 10 per cent. This classification offers very little indication of the risk of rejection of lots in which the quality is good.

It may be impossible to design a sampling plan to meet specifications of both AQL and LTPD for both n and c discrete. It is possible to approximate the values of Pa for the AQL and LTPD by the following method. A table or chart of the cumulative Poisson distribution is consulted to find c which most nearly meets the above specifications. Once the value of c is established for which the value of n of the PR level nearly equals the value of n of the CR level, the sampling plan is established and the values for the remaining points on the OC curve can be found from the cumulative Poisson distribution table or from equation (3). Tables have been developed by Cameron which aid in simplifying this procedure. (10, p. 37) This method would give some insight into both the consumer's risk and the producer's risk for a given sampling plan.

Another measure of the effectiveness of a sampling plan is the average outgoing quality curve. If lots rejected by a given sampling plan are detailed and cleared of defective items which are replaced by good items, the quality accepted is indicated by the average outgoing quality level. (11, p. 7) This is the average outgoing quality plotted against the incoming quality and illustrates the level of outgoing quality assuming 100 per cent inspection of rejected lots. An approximation to the AOQL curve is obtained for a selected plan by multiplying the ordinate of the OC curve by the corresponding abscissa value. (3, p. 310)

The average sample number (ASN) curve is an indication of the sampling required by the plan where the average sample size is plotted against the incoming quality level. This information is very useful when considering the inspection cost for any sampling. For single sampling plans the ASN is merely n, but for double and sequential sampling plans, the ASN is dependent upon the incoming quality level. Altman studies the ASN and AOQL to establish a relation between the sample size and the AOQL for single sampling plans. (12, p. 29) This study indicated that multiplying the sample size by a constant k reduced the AOQL to 1/k of its original value regardless of the acceptance number.

Another illustration of the effectiveness of a sampling plan over a range of incoming quality level is given by the Total Average Inspection (TAI) curve. If the rejected lots are returned to the supplier, the TAI is merely equal to ASN as described above. If the rejected lots are retained and inspected 100 per cent, the total amount of inspection will depend on the quality of material submitted. This curve is the total inspection plotted against the lot fraction defective. In single sampling plans the TAI can be computed from the following formula. (2, p. 138)

$$TAI = n + (1 - Pa)(N - n) \tag{5}$$

Where the value for Pa is obtained from the OC curve.

These methods of illustrating the effectiveness of a sampling plan provide little information as to the protection afforded by the plan. This protection, the ability to discriminate between good and bad lots, is illustrated in the OC curve. Freeman writes of the protection: (13, p. 4)

> There are clearly many possible plans of action and the problem arises how to choose among them. One important basis of choice is the amount of protection afforded by each plan of action, that is the relative frequency with which each plan of action will accept good lots and reject bad lots..... This information is contained in the OC curve.

A sampling plan might be considered ideal (100 per cent efficient) if it were able to discriminate perfectly between good and bad inspection lots. (6, p. 20)

It is generally recognized that increasing the sample size increases the ability of a plan to discriminate between lots of different qualities. (14, p. 322) This is illustrated in Fig. 2. There is no practical method of measuring this change in efficiency which results from either a change in sample size or a change in acceptance number as illustrated in Fig. 3. Goode indicates that much of the information required for making the best choice of slope (OC curve) is difficult or impossible to obtain. (15, p. 18)

The purpose of this thesis is to use a predetermined criterion for the evaluation of the efficiency of acceptance sampling plans to determine the effect of changing the sample size, acceptable quality level and acceptance number.

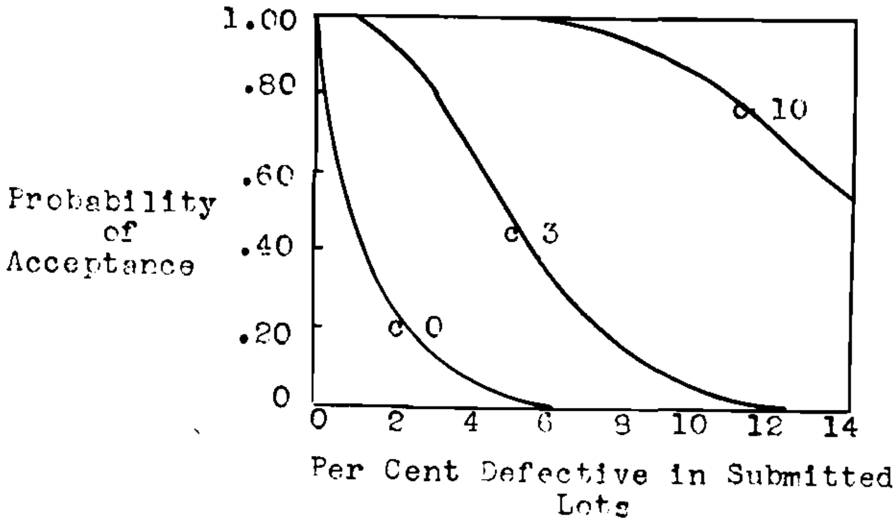

Fig. 3. OC Curve Showing Effect of Change in Acceptance Number (Constant Sample Size)

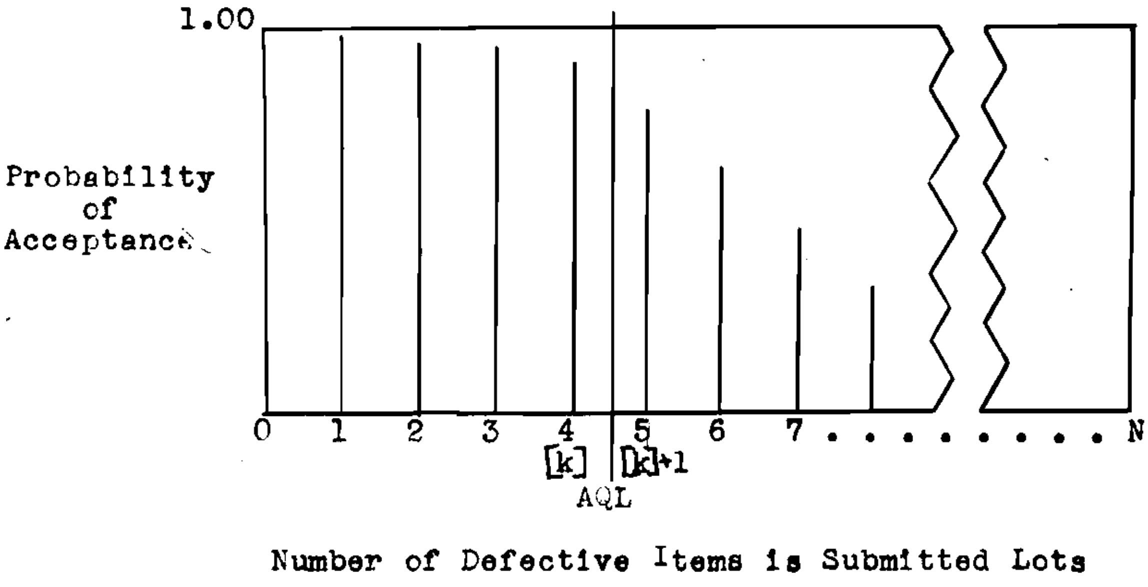

Fig. 4. OC Curve Showing Finite Points

CHAPTER II

DEFINITION OF EFFICIENCY

The efficiency of a sampling plan can be determined from the OC curve since it shows the probability that the sampling plan will accept lots whose fraction defective ranges from zero to one. All submitted lots whose fraction defective is less than or equal to the AQL should be accepted and all submitted lots whose fraction defective is greater than the AQL should be rejected. No sampling plan of less than 100 per cent inspection will provide this perfect discrimination.

There exist only a finite number of points on the OC curve. The number of these points and the corresponding ordinate values depends on the size of the inspection lot. If the lot contains N items, there exists only N + 1 points on the OC curve, since the number of defectives can only be 0, 1, 2 ..... N. The abscissa can then be expressed in terms on the number of defective items (m) in the lot, where m ranges from zero to N. This is illustrated in Fig. 4. For each discrete m there is a corresponding probability of acceptance which can be determined in terms of the hypergeometric distribution. Each of the probabilities of acceptance, in a perfect sampling plan, would be one where m is less than or equal to N(AQL) and zero where m is greater than N(AQL). This is not realizable under sampling conditions.

The producer's efficiency (Ep) of a sampling plan (a measure of the ability to accept lots which should be accepted) as defined in this thesis is the ratio of the sum of the probabilities of acceptance which correspond to the discrete values of m where m ranges from zero through N(AQL), to the sum of the corresponding probabilities of acceptance which would be possible if the sampling plan were perfect. Thus:

$$Ep = \frac{\sum_{m=0}^{[k]} \sum_{x=0}^{c} \frac{\binom{N-m}{n-x}\binom{m}{x}}{\binom{N}{n}}}{[k] + 1} \tag{6}$$

Where $[k]$ is the largest integral value for m which is less than or equal to the N(AQL).

The consumer's efficiency (Ec) of a sampling plan (a measure of the ability to reject lots which should be rejected) as defined in this thesis is the ratio of the sum of the probabilities of rejection; which correspond to the discrete values of m where m ranges from N(AQL) + 1 through N, to the sum of the corresponding probabilities of rejection which would be possible if the sampling plan were perfect. Thus:

$$Ec = \frac{\sum_{m=[k]+1}^{N} \sum_{x=c+1}^{n} \frac{\binom{N-m}{n-x}\binom{m}{x}}{\binom{N}{n}}}{N - [k]} \tag{7}$$

Where $[k]$ + 1 is the smallest integral value for m which is greater than N(AQL).

The efficiency (E) of a sampling plan (a measure of the ability to discriminate between good and bad lots) as defined in this thesis is the ratio of the sum of the probabilities of acceptance and the probabilities of rejection as described above to the sum of the corresponding probabilities which would be possible if the sampling plan were perfect. Thus:

$$E = \frac{\sum_{m=0}^{[k]} \sum_{x=0}^{c} \frac{\binom{N-m}{n-x}\binom{m}{x}}{\binom{N}{n}} + \sum_{m=[k]+1}^{N} \sum_{x=c+1}^{n} \frac{\binom{N-m}{n-x}\binom{m}{x}}{\binom{N}{n}}}{N+1} \tag{8}$$

CHAPTER III

PROCEDURE

With a very large N it is possible to approximate the values needed by equations (6), (7), and (8) by integration to determine the areas involved rather than values for specific points. This was performed by using the Poisson distribution which approximates the hypergeometric distribution as N becomes infinite. Thus the efficiency equation (8), becomes:

$$E = \int_0^{AQL} \left[\sum_{x=0}^{c} \frac{(np)^x \, e^{-np}}{x!} \right] dp + (1 - AQL) -$$

$$\int_{AQL}^{1} \left[\sum_{x=0}^{c} \frac{(np)^x \, e^{-np}}{x!} \right] dp \qquad (9)$$

Where the integration from zero to AQL is the area of correct action of accepting lots which should be accepted and the integration from AQL to one is the incorrect action area of accepting lots which should have been rejected.

The equation can be simplified by setting np equal to u.

Let $u = np$

$du = ndp$

$dp = du/n$

$$E = \sum_{x=0}^{c} \int_{0}^{nAQL} \frac{(u^x\ e^{-u})}{n\ x\ !}\ du + (1 - AQL) - \sum_{x=0}^{c} \int_{nAQL}^{1} \frac{(u^x\ e^{-u})}{n\ x\ !}\ du \quad (10)$$

To simplify the computations, it was useful to reduce the integration procedures as follows:

$$\int_{0}^{nAQL} \frac{u^x\ e^{-u}}{n\ x\ !}\ du = -\left[\frac{1}{n\ x\ !\ e^u} (u^x + x\ u^{x-1} + (x)\ (x-1)\ u^{x-2} + \ldots\ldots + x\ ! \right]_{0}^{nAQL} \quad (11)$$

so that

$$\int_{0}^{nAQL} \frac{u^x\ e^{-u}}{n\ x\ !}\ du = \frac{1}{n} - \left[\frac{1}{n\ x\ !\ e^{nAQL}} (nAQL)^x + x\ (nAQL)^{x-1} + x\ (x-1)\ (nAQL)^{x-2} \ldots\ldots \right. \quad (12)$$

This was further reduced by using

$$\frac{1}{n} \sum_{x=0}^{c} \frac{u^x\ e^{-u}}{x\ !} = \frac{1}{n\ e^u} \left(\frac{u^0}{0!} + \frac{u^1}{1!} + \frac{u^2}{2!} \ldots\ldots \frac{u^c}{c!} \right) \quad (13)$$

which becomes

$$\frac{1}{n} \sum_{x=0}^{c} \frac{u^x\ e^{-u}}{x\ !} = \frac{1}{n\ e^u\ x\ !} \left(u^x + x\ u^{x-1} + (x)\ (x-1)\ u^{x-2} \ldots\ldots\ x\ ! \right) \quad (14)$$

Substituting in equation (18)

$$\int_0^{nAQL} \frac{u^x e^{-u}}{n x !} du = \frac{1}{n} - \frac{1}{n} \sum_{x=0}^{c} \frac{u^x e^{-u}}{x !} \qquad (15)$$

The second integral of equation (10) was simplified as follows:

$$\int_{nAQL}^{n} \frac{(u^x e^{-u})}{n x !} du = - \frac{1}{n x ! e^u} (u^x + x u^{x-1} + (x)$$

$$\left. (x - 1) u^{x-2} + \ldots\ldots\ldots x !) \right]_{nAQL}^{n} \qquad (16)$$

So that

$$\int_{nAQL}^{n} \frac{(u^x e^{-u})}{n x !} du =$$

$$- \frac{1}{n x ! e^n} \left[n^x + nx^{x-1} + (x) (x - 1) N^{x-2} + \ldots\ldots + x ! \right] +$$

$$+ \frac{1}{n x ! e^u} \left[nAQL^x + (x) nAQL^{x-1} + (x) (x - 1) \right.$$

$$\left. (nAQL)^{x-2} + \ldots\ldots\ldots + x !) \right. \qquad (17)$$

Application of equation (14) produces

$$\int_{nAQL}^{n} \frac{u^x e^{-u}}{x ! n} du = \frac{1}{n} \sum_{x=0}^{c} \frac{u^x e^{-u}}{x !} \qquad (18)$$

Substituting equation (15) and equation (18) in equation (10) producing the following result which was used in this thesis:

$$E = \frac{1}{n} - \sum_{x=0}^{c} \frac{1}{n} \sum_{x=0}^{c} \frac{u^x e^{-u}}{x!} +$$

$$(1 - AQL) - \sum_{x=0}^{c} \frac{1}{n} \sum_{x=0}^{c} \frac{u^x e^{-u}}{x!} \qquad (19)$$

It is possible to restate the producer's efficiency, equation (6) and the consumer's efficiency equation (7), as follows:

$$Ep = \frac{\int_0^{nAQL} \frac{1}{n}\left[\sum_{x=0}^{c} \frac{u^x e^{-u}}{x!}\right] du}{AQL} \qquad (20)$$

Where the numerator is the area below the OC curve from zero to AQL and the denominator is the total area to the left of AQL.

and

$$Ec = \frac{(1-AQL) - \int_{nAQL}^{n} \frac{1}{n}\left[\sum_{x=0}^{c} \frac{u^x e^{-u}}{x!}\right] du}{1 - AQL} \qquad (21)$$

Where the numerator is the area above the OC curve from AQL to one and the denominator is the total area to the right of AQL.

Application of equations (11) through (18) produced the following results which were used in this thesis

$$Ep = \frac{\frac{1}{n} - \sum_{x=0}^{c} \frac{1}{n} \sum_{x=0}^{c} \frac{u^x e^{-u}}{x!}}{AQL} \tag{22}$$

$$Ec = \frac{(1 - AQL) - \sum_{x=0}^{c} \frac{1}{n} \sum_{x=0}^{c} \frac{u^x e^{-u}}{x!}}{1 - AQL} \tag{23}$$

The values for the summation terms were obtained from Molina's Tables which gives the probability of x occurrences at least c times in n trials. (16, Table II) Therefore, the probability of x occurring c or fewer times in n trials (Pa) can be obtained by the following equation.

$$Pa = 1 - \sum_{x = c+1}^{\infty} \frac{u^x e^{-u}}{x!} \tag{24}$$

The sample sizes selected were 20, 40, 60, 80, 100, 120, 300, 400, 500, 600, 800, and 1000. The values for c for each n ranged from zero to a value comparable to the corresponding value in Military Standard 105A. (17) The AQL values from .5 per cent to 10 per cent were selected in the same incremental steps as a Military Standard 105A. This permits analysis of the efficiency of a range of sampling plans illustrated in Military Standard 105A. These procedures and tables are discussed in many of the leading textbooks on Quality Control and are widely used in industry. (2, p. 175)

The results of the calculations are presented in Chapter III.

The Poisson distribution as used above is an approximation to the hypergeometric distribution. To justify the use of the Poisson distribution to determine the efficiency, the efficiency was determined for a few sampling plans by the use of an approximation to the hypergeometric distribution. The points on the OC curve were determined by the use of f binomial distribution which gives a good approximation to the hypergeometric where p is very small. Probability values for the f binomial were determined directly from the Binomial tables by using the following substitutions:

Symbol in f binomial	M	f	x
Symbol for entering table	n	p	x

Where n equals sample size, f equals n divided by N, the lot size, and M equals pn. (18, p. XIV)

For these test calculations, tables of the cumulative binomial distribution were used to determine the Pa for changes in p of 1 per cent. (5) The incremental areas were determined from these values and added. The correct action areas were determined and the efficiency established by the definition in Chapter II. Lot sizes of 200 and 1000 were chosen with the sample size ranging from 40 to 120 in steps of 20 and the AQL was selected as 1 per cent. The results of these computations are presented in Table 1.

CHAPTER IV

RESULTS

The results in Table 1 present the efficiencies of various sampling plans computed by the approximation to the hypergeometric distribution and by the Poisson distribution. The Poisson distribution assumes an inspection lot which is large in comparison with the sample size. Lot sizes of 200 and 1000 were used with the approximation to the hypergeometric distribution. AQL for all plans in this table is 1 per cent.

	N	c	n 40	n 60	n 80	n 100	n 120
Hypergeometric	200	1	96.11	97.79	98.61	99.04	
Hypergeometric	1000	1	96.02	97.60	98.36	98.36	99.04
Poisson		1	95.95	97.58	98.35	98.79	99.04
Hypergeometric	200	2	93.75	96.26	97.30	98.24	
Hypergeometric	1000	2	93.58	96.03	97.24	97.98	98.44
Poisson		2	93.48	95.99	97.22	97.97	98.34
Hypergeometric	200	3	91.52	94.57	96.04	97.25	
Hypergeometric	1000	3	91.03	94.38	96.03	97.04	97.69
Poisson		3	90.98	94.33	95.99	95.99	97.65

Table 1. Comparison of Efficiencies of Sampling Plans With AQL of 1 Per Cent Showing the Influence of Lot Size

The efficiencies calculated over this range are in close agreement. The largest difference in this range is 0.54 per cent where c is 3 and the sample size and lot size are small. One reason for the close agreement is the errors incurred with a small lot size tend to cancel out when the efficiency is computed. The greatest difference would be in consumers or producers efficiency. This indicates that a seperate study of small lot sizes would be most profitable.

The practical aspects of acceptance sampling indicate that the lot size should be as large as possible. (6, p. 42)

> The OC curve of a sampling plan depends primarily on the number of items inspected per inspection lot; the larger this number, the better the protection that the sampling plan gives against the rejection of high-quality lots. But the total cost of inspection depends primarily on the <u>percentage</u> of the submitted items that are inspected..... Since we want both a large number of items in the sample (for good protection) and a small percentage of items (for low cost) it follows that large inspection lots are desirable.

Thus the results calculated by the Poisson distribution; which assumes a large inspection lot, should be useful.

The results of the calculations using equations (19), (22) and (23) are presented in Figs. 5 through 18. The efficiencies presented in Fig. 5 are for an AQL of 5 per cent. Fig. 6 presents the producer's and consumer's efficiencies for sampling plans with the same AQL. This procedure, presenting the efficiency for a given AQL followed by the Producer's and Consumer's efficiency for the same AQL, is continued for all given values of AQL.

The acceptance numbers are presented in increments to produce 12 to 15 lines per Figure. This was required if the large amounts of data were to be presented in useable form. Linear interpolation for any intermediate numbers will produce an approximation which should be exact enough for practical use. An envelop across the maximum efficiencies will assist interpolation in the efficiency Figures where the lower acceptance number has a decreasing efficiency.

Decreasing efficiency for a low acceptance number; such as zero, with an increasing sample size, indicates that the OC curve is changing to the left of the AQL. This is illustrated by the rate at which the producer's efficiency decreases in this range.

Use of the results of this thesis can be illustrated as follows:

1. Given an AQL, it is possible to determine a sampling plan which will produce the desired efficiency.
2. Given a sampling plan, it is possible to determine how much the efficiency is affected by a change in sample size, acceptance number or acceptable quality level.
3. Given a sampling plan, it is possible to determine how the risks are divided between producer and consumer.

The purpose of this thesis is therefore accomplished since the evaluation of acceptance sampling plans can be determined from the results in Figs. 5 through 18. However, it must be emphasized that the definition of efficiency as used in this thesis is based on the concept that the per cent defective of a submitted lot is unknown and likely to be any value from zero to one.

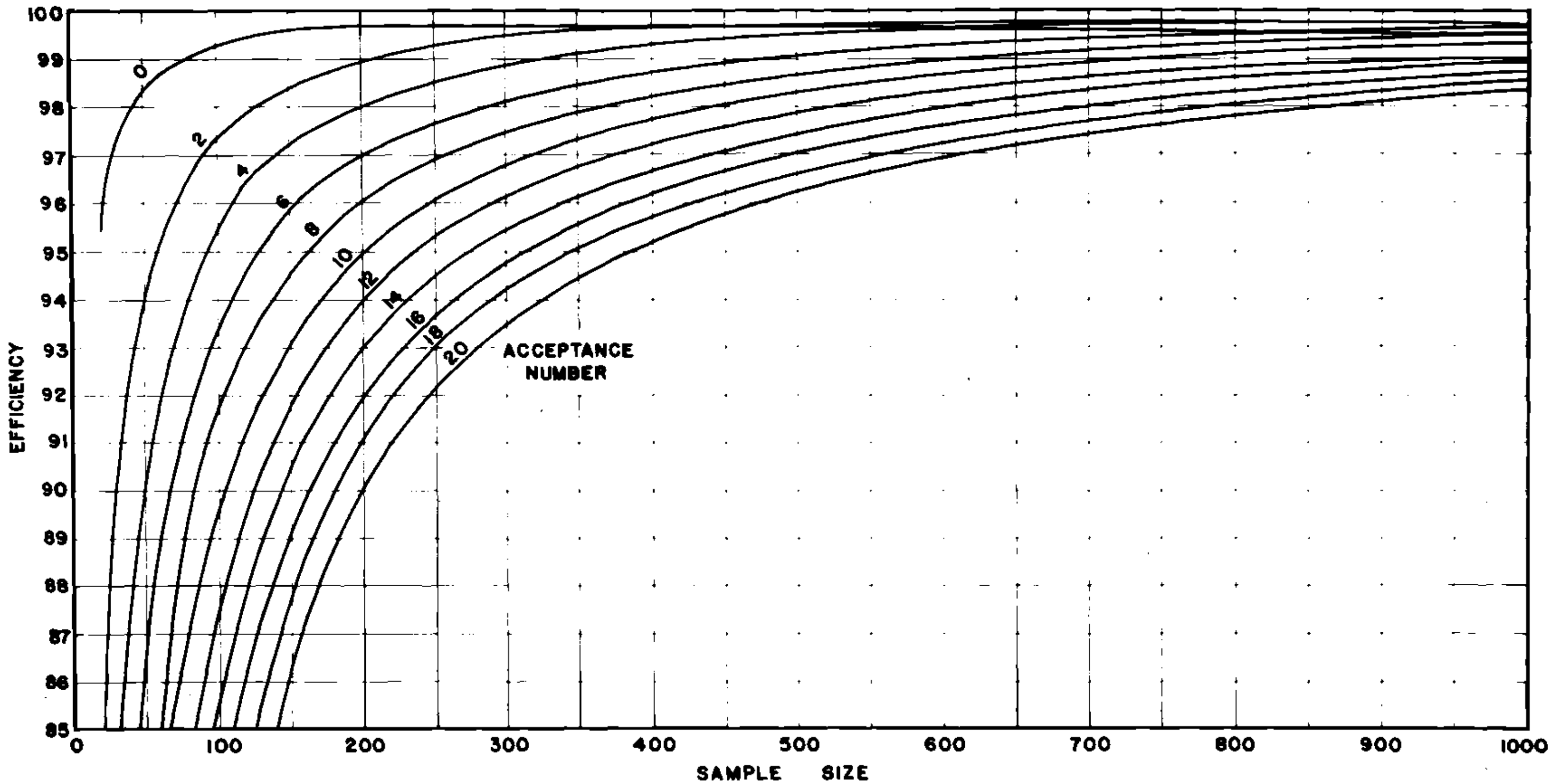

FIG. 5 EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF .5 PER CENT.

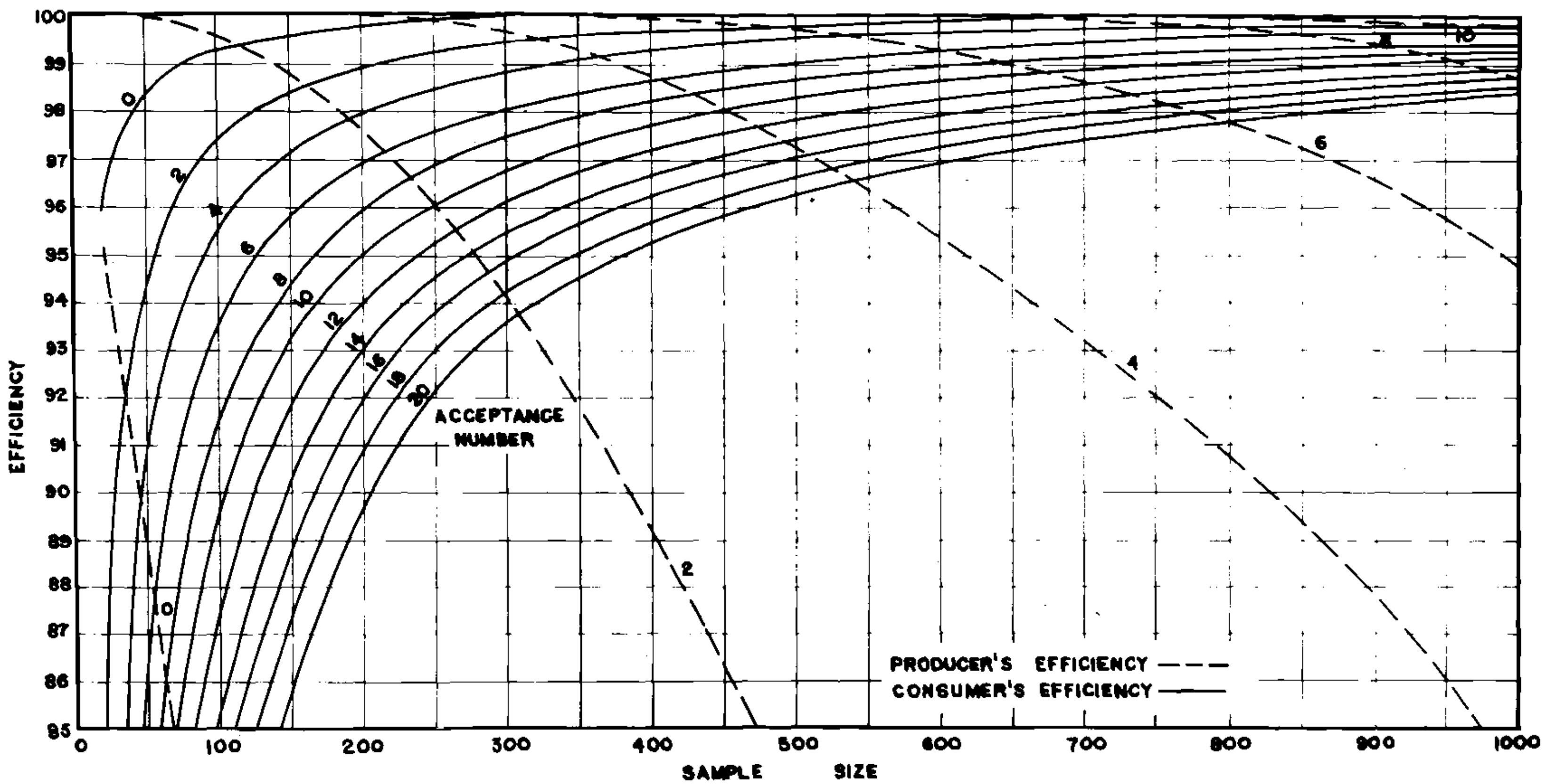

FIG. 6 PRODUCER'S & CONSUMER'S EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF .5 PER CENT

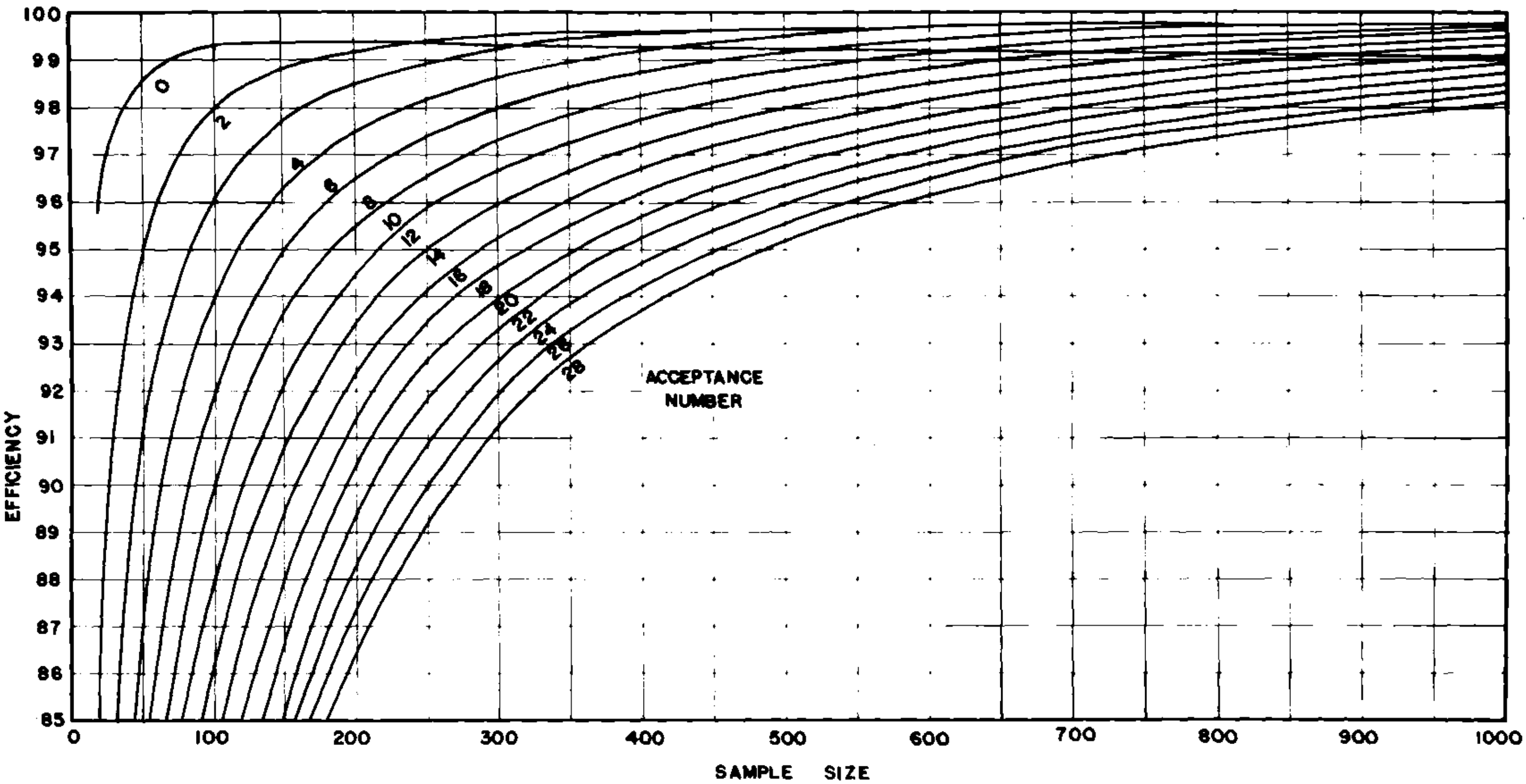

FIG. 7 EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 1 PER CENT

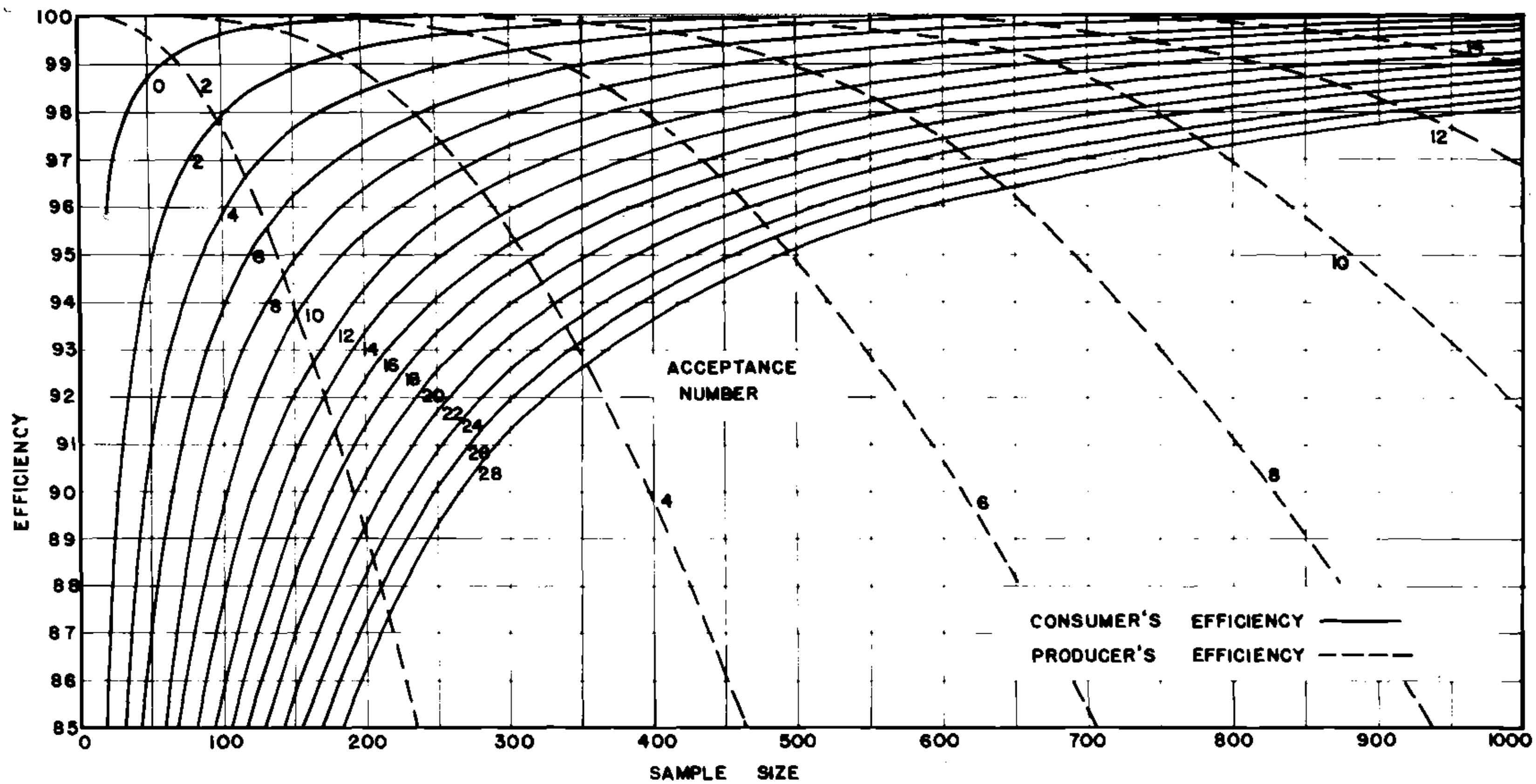

FIG. 8 PRODUCER'S & CONSUMER'S EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 1 PER CENT

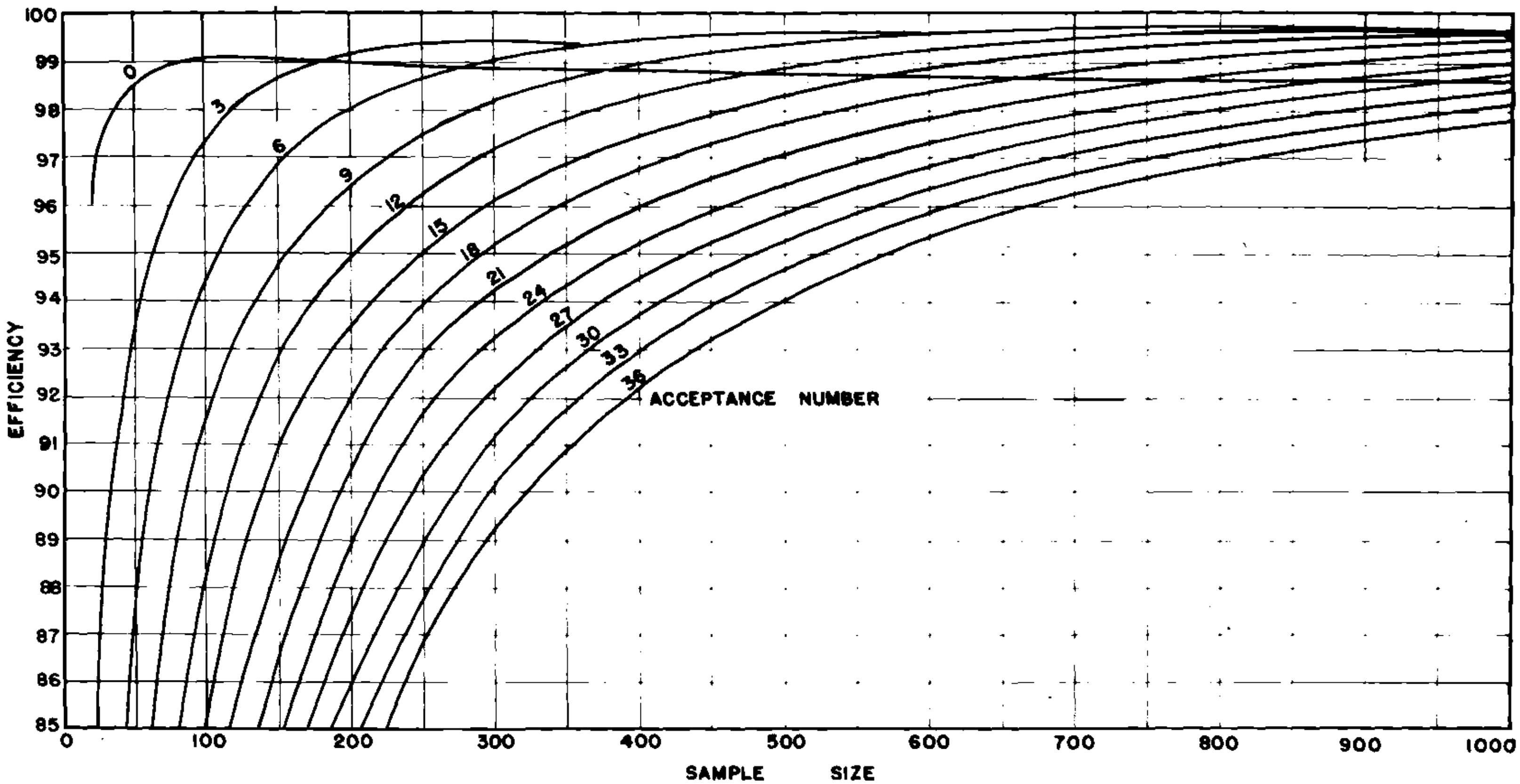

FIG. 9 EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 1.5 PER CENT.

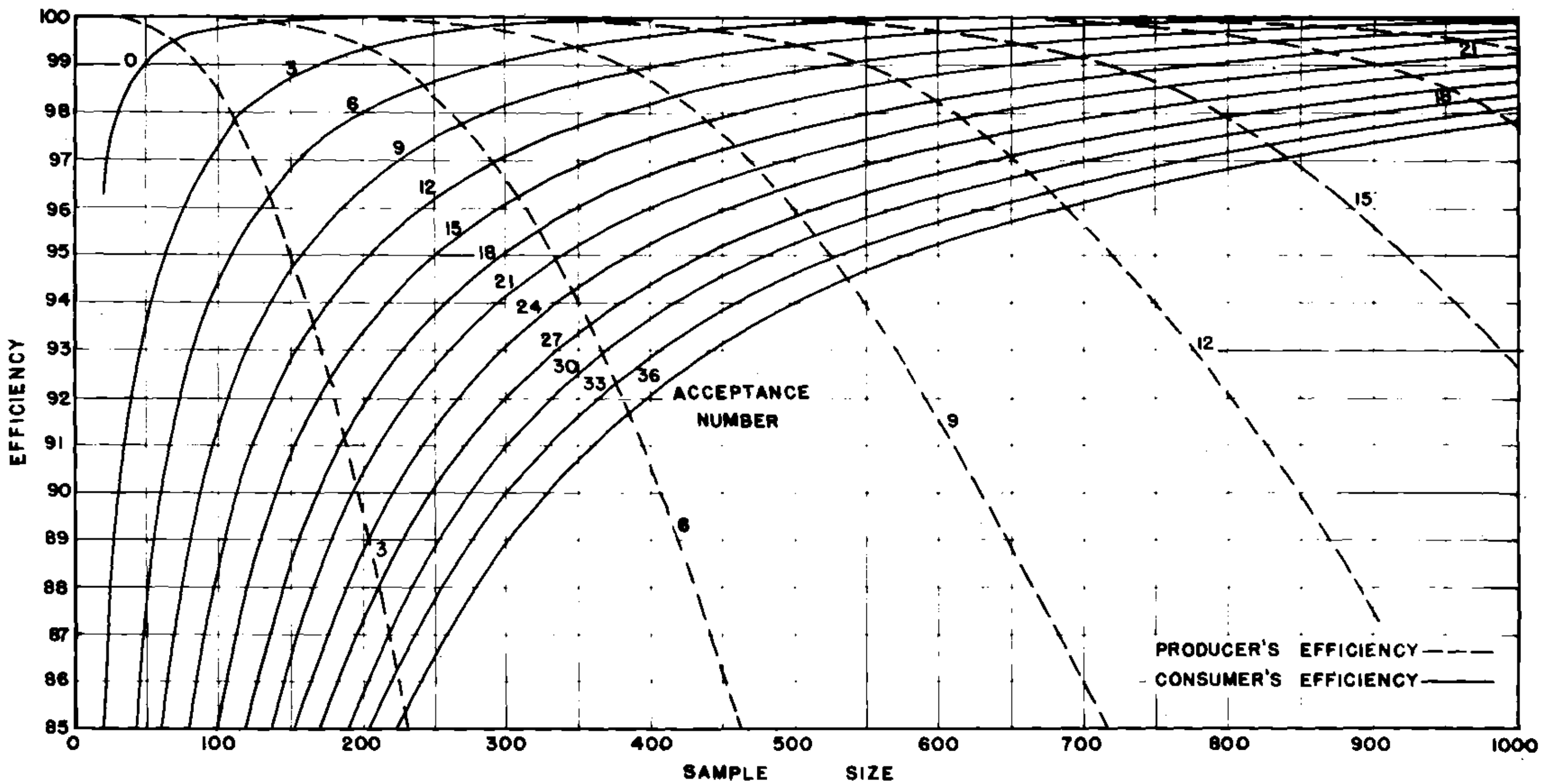

FIG. 10 PRODUCER'S & CONSUMER'S EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 1.5 PER CENT.

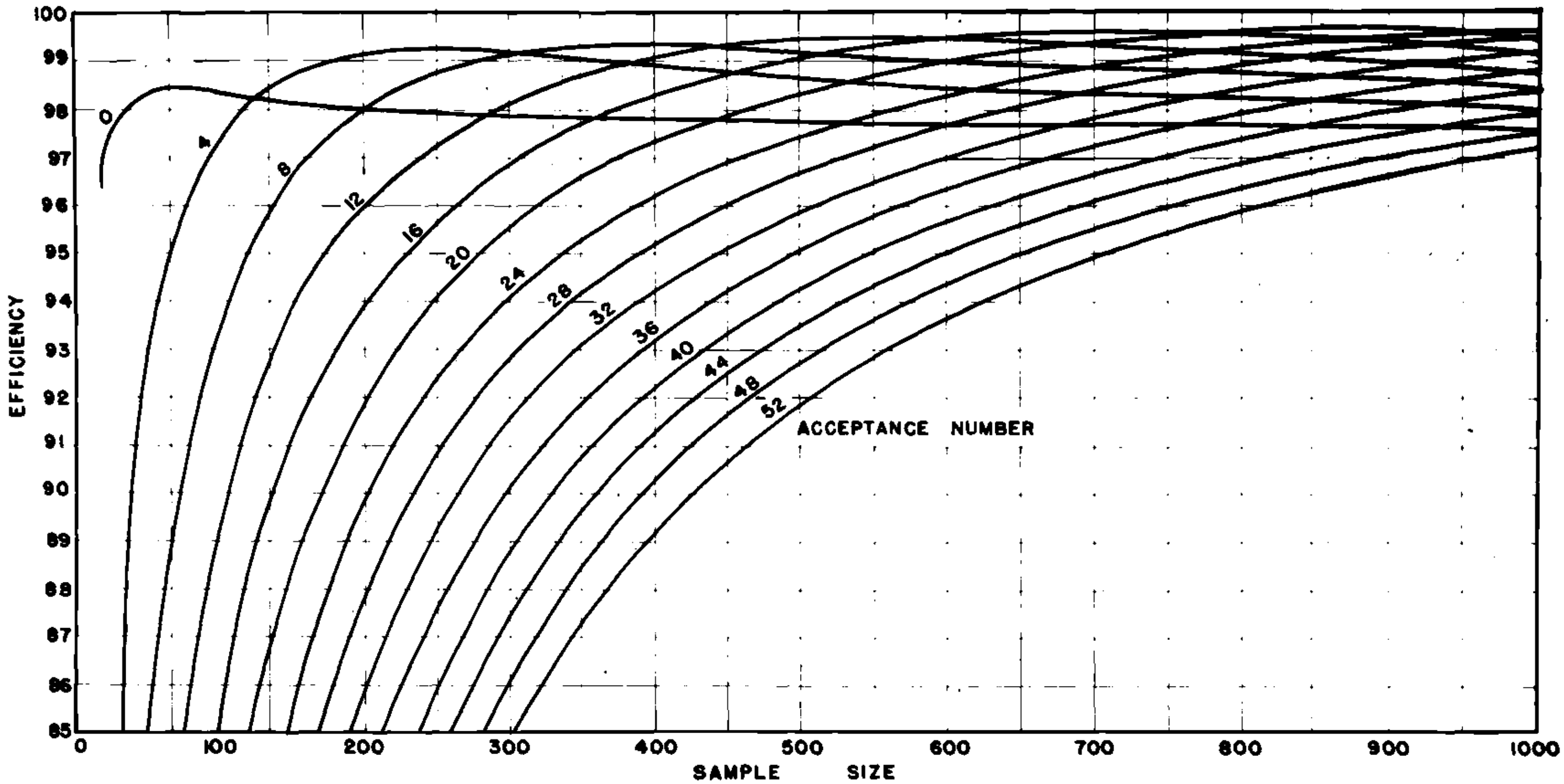

FIG. 11 EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 2.5 PER CENT.

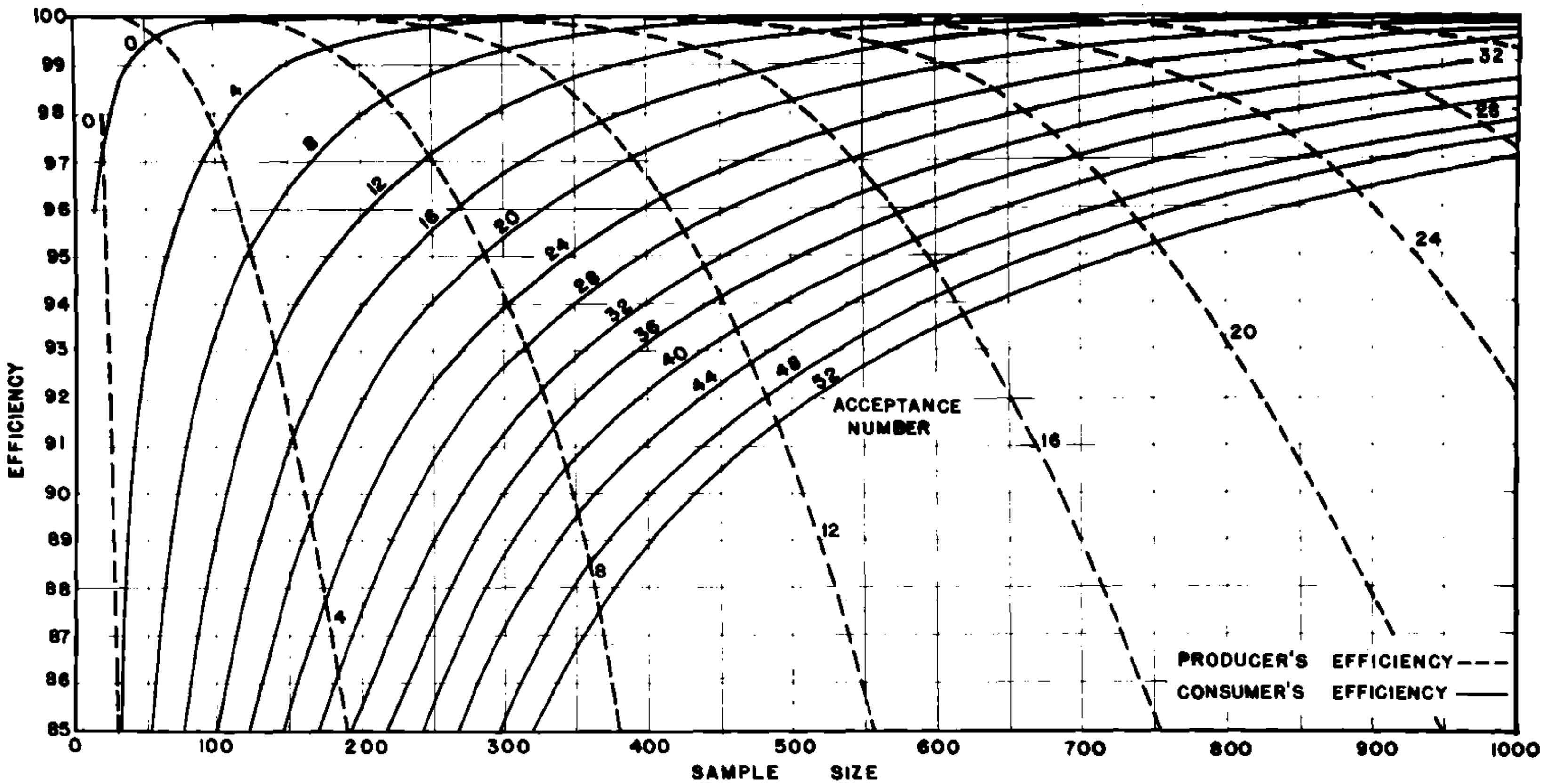

FIG. 12 PRODUCER'S & CONSUMER'S EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 2.5 PER CENT.

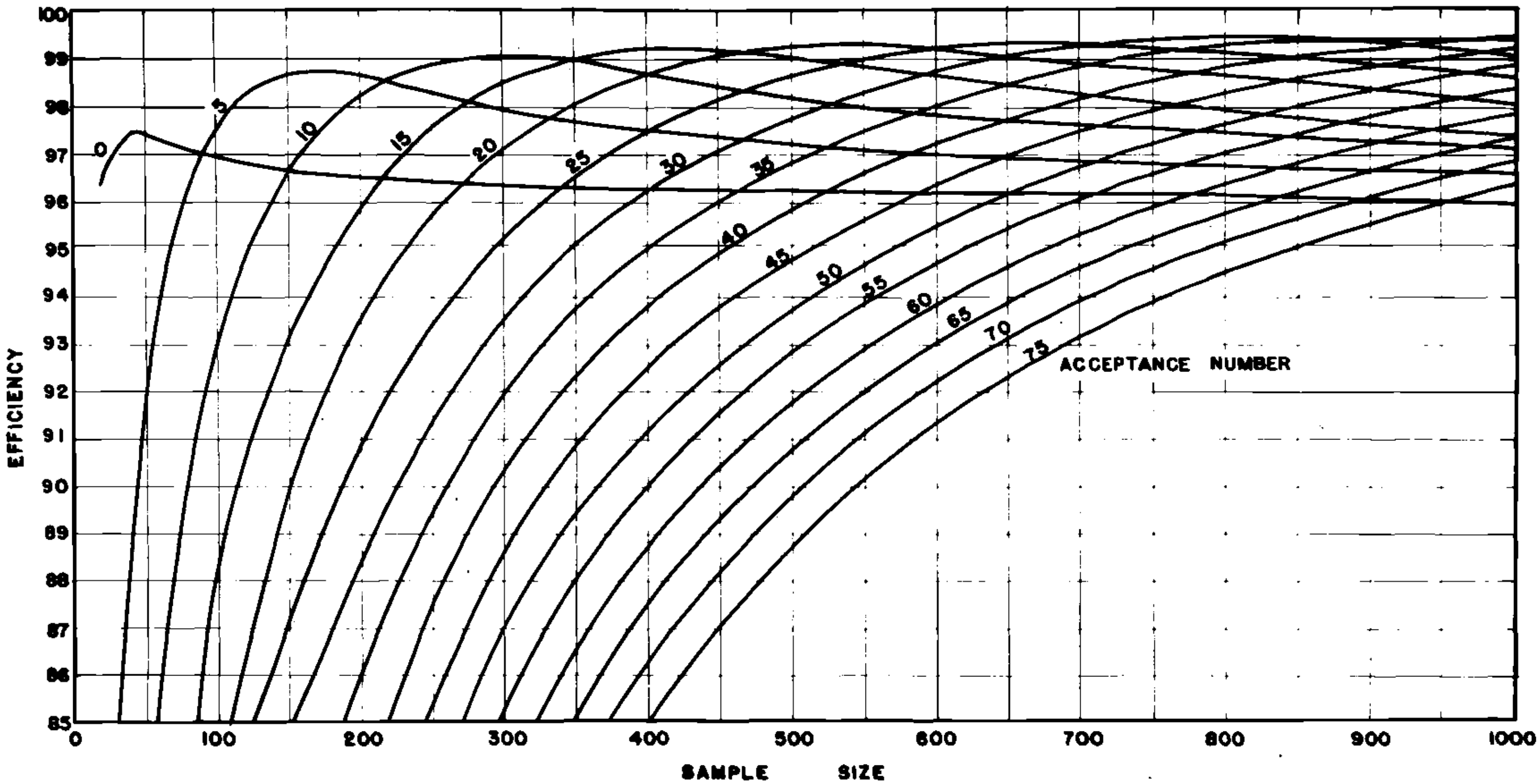

FIG. 13 EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 4 PER CENT.

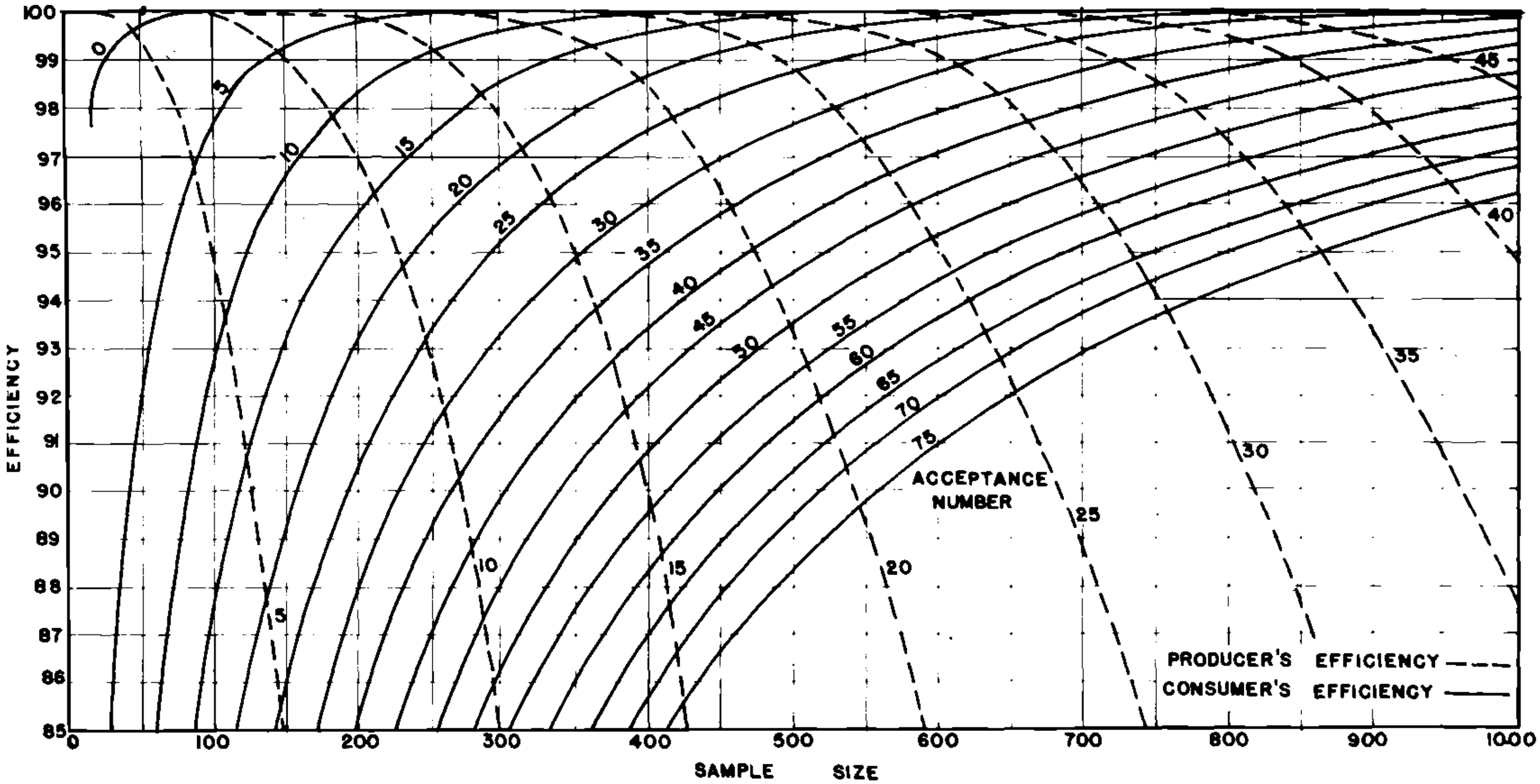

FIG. 14 PRODUCER'S & CONSUMER'S EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 4.0 PER CENT.

[redacted]

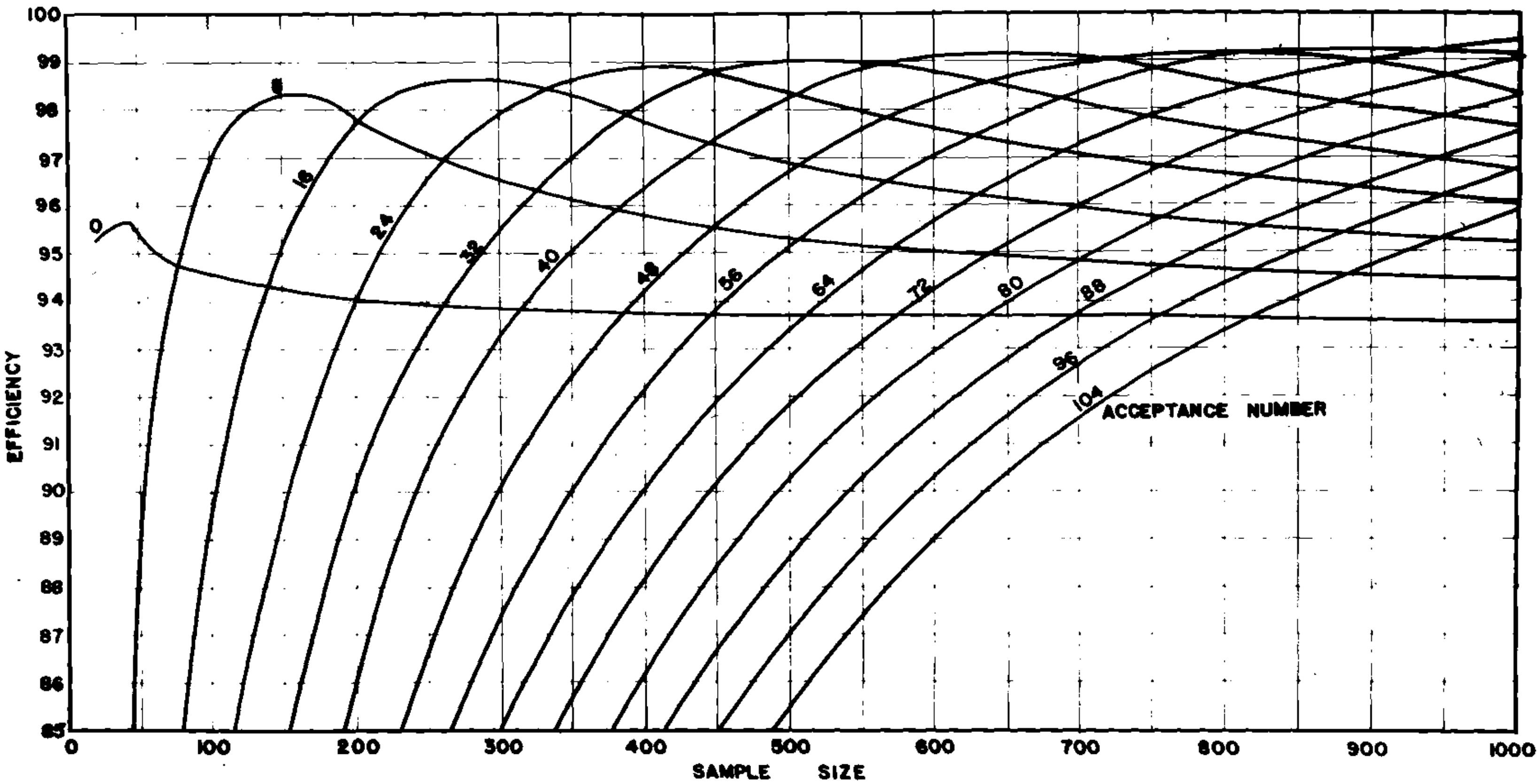

FIG. 15 EFFICIENY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 6.5 PER CENT.

[redacted]

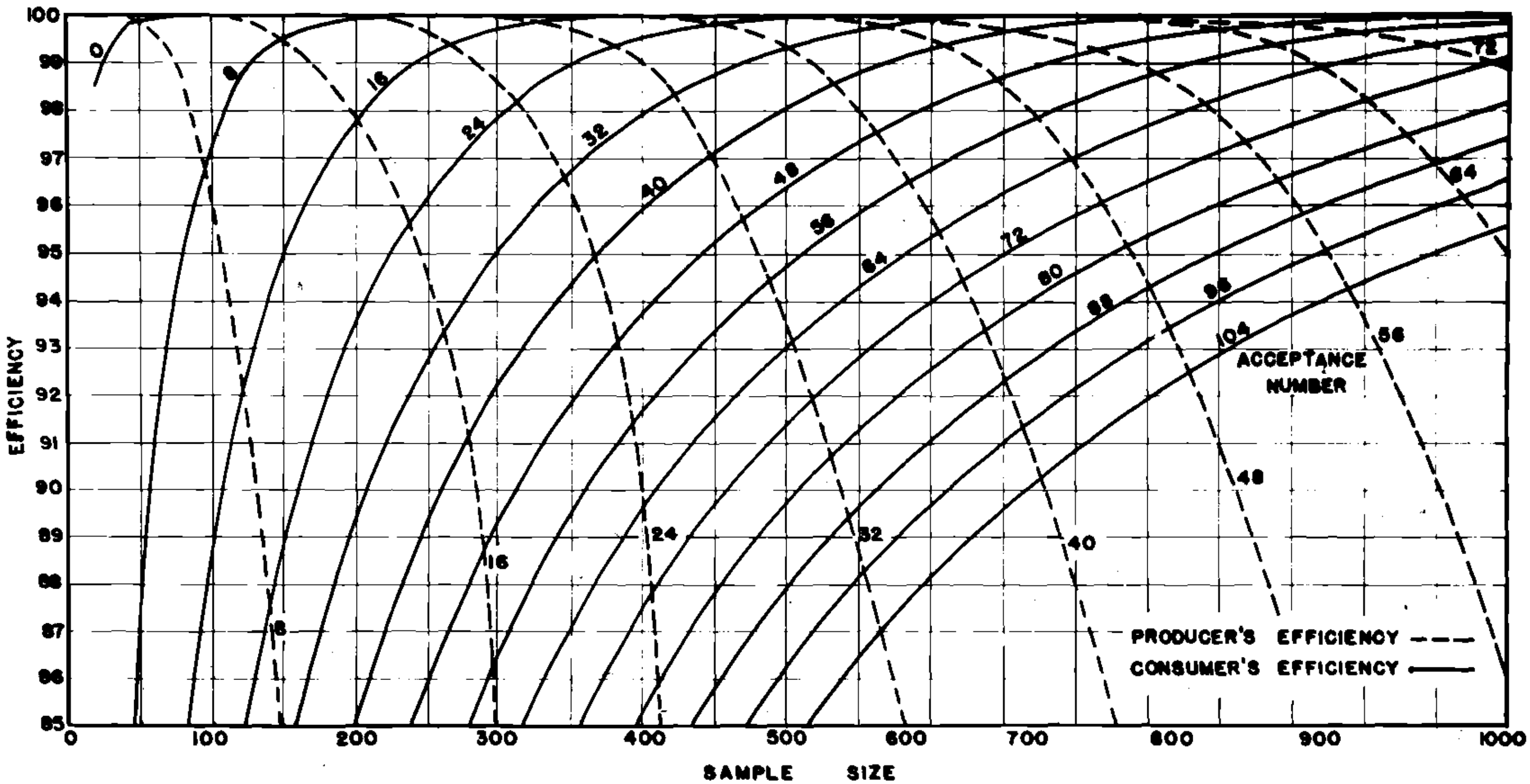

FIG. 16 PRODUCER'S & CONSUMER'S EFFICIENCY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 6.5 PER CENT.

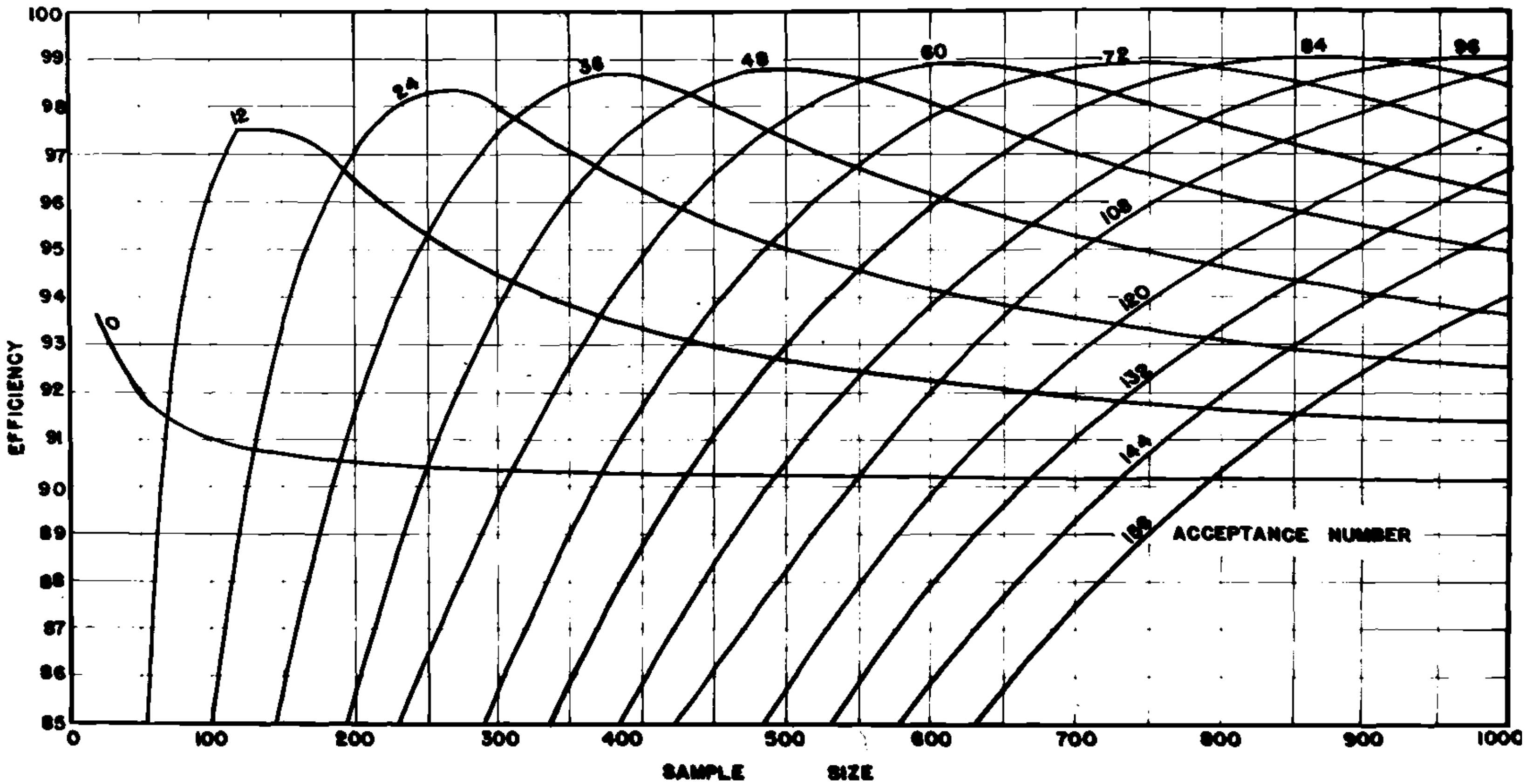

FIG. 17 EFFICIENY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 10 PER CENT.

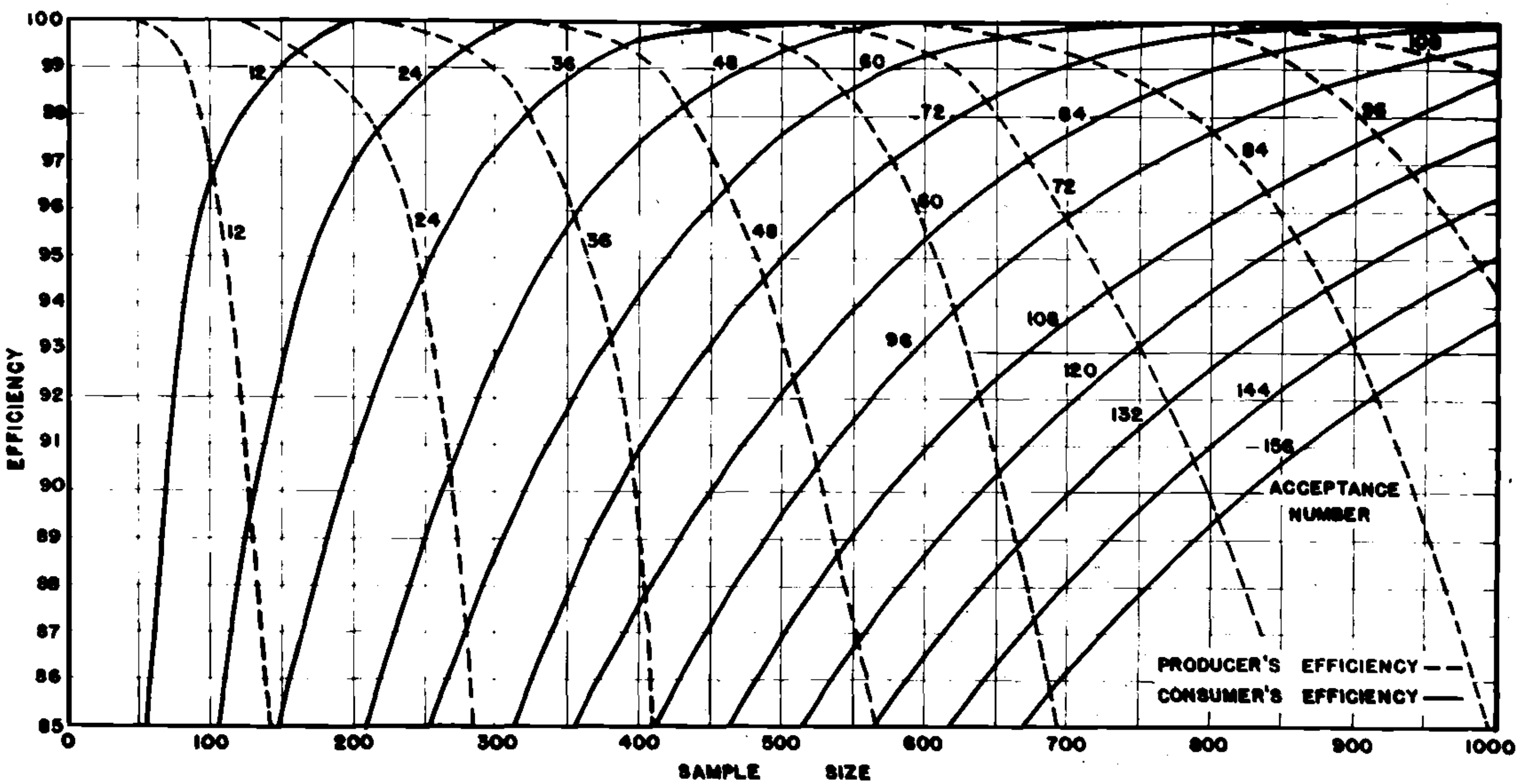

FIG. 18 PRODUCER'S & CONSUMER'S EFFICIENY OF SAMPLING PLANS WITH ACCEPTABLE QUALITY LEVEL OF 10 PER CENT.

CHAPTER V

CONCLUSIONS

The following conclusions have been reached from the results obtained in this work:

1. The efficiency of a sampling plan does not increase in proportion to an increase in sample size. The change in efficiency resulting from changes in sample size can be determined from the results of this thesis.

2. Where the acceptable quality level is defined as the per cent defective in submitted lots at which 95 per cent of these lots will be accepted and this definition is maintained, the producer's efficiency is always higher than the consumer's efficiency at less than 100 per cent inspection.

3. Where the acceptable quality level is not dependent on the above definition and this acceptable quality level is held constant, an increase in sample size results in an increase in consumer's efficiency and a decrease in producer's efficiency.

CHAPTER VI

RECOMMENDATIONS

The use of the Poisson distribution in the computations did not seriously impair the results of the thesis for two reasons; one, the Poisson distribution can be considered as an approximation to the hypergeometric distribution where the lot size is large in relation to the sample size and two, it is advisable to assemble large inspection lots to achieve the lowest inspection cost per unit. Yet this is not always possible under actual conditions. Therefore, it is recommended that an analysis of the efficiency of sampling plans be conducted using the hypergeometric distribution. This would be most useful in the analysis of sampling from small lots.

The scientific approach to quality control demands the utilization of all available information when selecting a sampling plan. The results of this thesis should provide information to assist in this selection but full utilization will become possible only when the relation between efficiency and cost are established. It is recommended that a program be established to determine this relationship for all cost to both producer and consumer. This study should involve both tangible and intangible cost which could result from all possible efficiencies.

It is also recommended that a study be conducted to determine the efficiency of sampling plans when the samples are taken from a process which is producing items whose quality is around the acceptable quality level.

Another area for consideration is a study similar to this thesis but using acceptance sampling by variables.

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